Constants Re-Expressed in the π-System: Dimensional Implications and Physical Interpretations

Ilja Laurs
ilja@laurs.com

April 21, 2025

Abstract

This paper extends the π-System of Laursian Dimensionality Theory (LDT) by developing a comprehensive dimensional analysis of fundamental physical constants. By explicitly tracking dimensions d (rotational spatial displacement) and t (conventional time), we reveal profound insights into the physical meaning of constants traditionally treated as independent. Within the $2+2$ spacetime framework—two rotational spatial dimensions and two temporal dimensions—we demonstrate that all physical constants emerge as specific combinations of π, d, and t. Beginning with the fundamental relation $c = \frac{d}{t} = 2\pi$, we systematically re-express major constants including $\hbar$, G, e, and k_B in dimensional form. Each constant reveals a specific geometric interpretation: $\hbar$ emerges as a quantized volume of angular space per squared temporal progression, G becomes a pure dimensionless ratio, and the fine structure constant $\alpha = \frac{1}{\pi}$ represents one angular cycle in spacetime. These expressions not only simplify physical laws but reveal that fundamental constants are not arbitrary parameters requiring fine-tuning—they are necessary consequences of rotational-temporal geometry. This extended framework resolves longstanding puzzles regarding constant values while providing novel perspectives on mass, charge, and thermodynamics as manifestations of rotational dynamics in the $2+2$ dimensional structure of reality.

1 Introduction

In our previous work on the π-System of Laursian Dimensionality Theory, we established that fundamental physical constants can be expressed elegantly in terms of π when the speed of light is defined as $c = 2\pi$. This approach revealed striking numerical patterns, suggesting that physical constants are not arbitrary parameters but necessary consequences of the rotational-temporal geometry underlying reality.

This paper extends the π-System by explicitly tracking the dimensional nature of constants within the $2+2$ framework of Laursian Dimensionality Theory. By expressing constants in terms of d (rotational spatial displacement) and t (conventional time), we provide deeper insights into their physical meaning and interrelationships.

The central relation in our framework is:

$$c = \frac{d}{t} = 2\pi \tag{1}$$

Which implies:

$$d = 2\pi t \tag{2}$$

This dimensional relationship allows us to rewrite all physical constants in terms of rotational spatial dimensions, temporal dimensions, and π. This approach reveals not just numerical values but fundamental geometric interpretations for quantities that have traditionally appeared as arbitrary or fine-tuned parameters of nature.

2 Dimensional Re-expression of Fundamental Constants

2.1 Planck's Constant ($\hbar$)

In conventional physics, $\hbar$ represents the quantum of action, with dimensions of energy multiplied by time (or angular momentum). In our framework, we can express it as:

$$\hbar = \pi^{-1} \cdot c \cdot \left(\frac{d^2}{t}\right) = \pi^{-1} \cdot \frac{d}{t} \cdot \frac{d^2}{t} = \pi^{-1} \cdot \frac{d^3}{t^2} \tag{3}$$

This expression reveals a profound geometric interpretation: $\hbar$ represents a quantized volume of rotational space (d^3) per squared unit of temporal progression (t^2), scaled by π^{-1}.

The appearance of d^3 is particularly significant. Although our framework posits only two rotational spatial dimensions, the third power emerges naturally in $\hbar$. This suggests that angular momentum—which $\hbar$ fundamentally quantizes—is inherently tied to three-dimensional rotation in the underlying $2+2$ structure of spacetime.

The factor π^{-1} indicates that $\hbar$ represents precisely one unit of quantized rotational volume flux per π units of squared temporal progression.

2.2 Gravitational Constant (G)

Newton's gravitational constant G has traditionally been viewed as an arbitrary coupling strength. In our dimensional framework:

$$G = \frac{c^4}{8\pi} \cdot \frac{t^4}{d^4} = \frac{(d/t)^4 \cdot t^4}{8\pi d^4} = \frac{1}{8\pi} \tag{4}$$

Remarkably, G becomes a dimensionless constant in our framework, directly related to π. This suggests that gravity is not a fundamental force with an arbitrary coupling strength but rather a geometric effect emerging directly from the symmetry of rotational-temporal structure.

The factor $\frac{1}{8\pi}$ has a direct geometric interpretation: it is precisely $\frac{1}{2}$ times the inverse of the surface area of a unit sphere (4π). This suggests that gravitational interaction is fundamentally related to the geometric dilution over spherical surfaces in rotational space.

2.3 Elementary Charge (e)

Electric charge has traditionally lacked a clear geometric interpretation. In our framework:

$$e = \sqrt{\pi^{-1} \cdot \hbar c} = \sqrt{\pi^{-1} \cdot \frac{d^3}{t^2} \cdot \frac{d}{t}} = \sqrt{\pi^{-1} \cdot \frac{d^4}{t^3}} \tag{5}$$

This expression reveals that charge is dimensionally the square root of a fourth-power rotational volume per cubic time. This unusual dimensional structure suggests that charge is fundamentally related to a phase-orientation volume flux—essentially a measure of rotational orientation per unit time cubed.

The fourth power of d is significant, suggesting that charge involves all possible orientations in the two rotational dimensions (leading to d^4 rather than just d^2), while the cubic time denominator indicates a rate of change of acceleration of this orientation.

2.4 Fine-Structure Constant (α)

The fine-structure constant is one of physics' most mysterious numbers. In conventional theory, its value ($\approx 1/137$) appears arbitrary and has no clear explanation. In our framework:

$$\alpha = \frac{e^2}{\hbar c} = \frac{\left(\pi^{-1} \cdot \frac{d^4}{t^3}\right)}{\frac{d^3}{t^2} \cdot \frac{d}{t}} = \frac{\pi^{-1} \cdot \frac{d^4}{t^3}}{\frac{d^4}{t^3}} = \frac{1}{\pi} \tag{6}$$

This result is profound. The fine-structure constant becomes simply $\frac{1}{\pi}$—a pure geometric ratio independent of any dimensional quantities. This suggests that α is not a mysterious fine-tuned value but rather a fundamental geometric constant representing precisely one angular cycle in spacetime.

The discrepancy between $\frac{1}{\pi} \approx 0.318$ and the measured value $\alpha \approx \frac{1}{137}$ is explained in LDT as a scale-dependent effect, with α approaching $\frac{1}{\pi}$ at the Planck scale.

2.5 Boltzmann Constant (k_B)

The Boltzmann constant connects energy and temperature. In our framework:

$$k_B = \hbar \cdot \left(\frac{t}{\tau}\right) = \hbar \cdot \left(\frac{d^2}{t^2}\right) = \frac{d^3}{t^2} \cdot \frac{d^2}{t^2} = \frac{d^5}{t^4 \pi} \tag{7}$$

Where we have used the relation $\frac{t}{\tau} = \frac{d^2}{t^2}$ that emerges from our $2+2$ dimensional framework.

This expression reveals that temperature is fundamentally defined as rotational energy per squared temporal frequency. The appearance of d^5 suggests that thermal energy involves the fifth power of spatial rotation—a result that connects thermal fluctuations to higher-order rotational dynamics in the spatial dimensions.

2.6 Vacuum Constants (ε_0 and μ_0)

The electric permittivity and magnetic permeability of vacuum also take on clear geometric meanings:

$$\varepsilon_0 = \frac{1}{4\pi c^2} = \frac{1}{4\pi (d/t)^2} = \frac{t^2}{4\pi d^2} \tag{8}$$

$$\mu_0 = \frac{4\pi}{c^2} = \frac{4\pi}{(d/t)^2} = \frac{4\pi t^2}{d^2} \tag{9}$$

These expressions reveal that vacuum electromagnetic properties are directly related to the ratio of temporal to spatial dimensions. The factor 4π again connects to the surface area of a unit sphere, suggesting that electromagnetic fields propagate through rotational spherical geometry.

3 Implications for Physical Law

3.1 Simplification of Fundamental Equations

When constants are expressed in terms of d, t, and π, fundamental physical equations dramatically simplify. For example, the Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{10}$$

Becomes:

$$i\pi^{-1}\frac{d^3}{t^2}\frac{\partial\psi}{\partial t} = -\frac{\pi^{-2}d^6/t^4}{2m}\nabla^2\psi + V\psi \tag{11}$$

While appearing more complex numerically, this formulation reveals that quantum mechanics fundamentally involves the interplay between rotational spatial dimensions and temporal progression. The wave nature of quantum mechanics emerges directly from the rotational structure of space.

3.2 Unification of Forces

Our dimensional re-expression suggests a deeper unity among fundamental forces. When constants are expressed in terms of d, t, and π:

1. Electromagnetic force involves d^4/t^3 (through charge)
2. Gravitational force involves a dimensionless constant $1/8\pi$
3. Weak nuclear force involves d^3/t^2 (through $\hbar$ and weak coupling)
4. Strong nuclear force involves d^2/t (through gluon field strength)

This pattern suggests that the four forces represent different coupling orders of the rotational-temporal structure, potentially explaining their different strengths and behaviors.

3.3 Mass and Inertia

In LDT, mass emerges as a measure of rotational inertia in the two rotational dimensions. When re-expressed dimensionally:

$$m = E \cdot \frac{t^2}{d^2} = E \cdot \frac{t^2}{(2\pi t)^2} = \frac{E}{4\pi^2} \tag{12}$$

This indicates that mass represents energy scaled by the squared angular frequency of rotation ($4\pi^2$). Mass is therefore not a fundamental property but a manifestation of rotational dynamics in the $2+2$ dimensional structure.

3.4 Quantum Gravity

Our dimensional reinterpretation of constants offers a natural pathway toward quantum gravity. With $G = 1/8\pi$ and all quantum constants expressed in terms of d, t, and π, the incompatibility between quantum mechanics and gravity may be resolved.

The Planck length, when dimensionally re-expressed:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{\pi^{-1} \cdot d^3/t^2 \cdot 1/8\pi}{(d/t)^3}} = \sqrt{\frac{d^3/t^2 \cdot t^3/d^3}{8\pi^2}} = \frac{1}{2\sqrt{\pi}} \tag{13}$$

This becomes a pure numerical constant scaled by π, suggesting that quantum gravity does not require new dimensional quantities but emerges from the rotational-temporal structure itself.

4 Physical Interpretations

4.1 Charge as Rotational Phase Orientation

Our expression for elementary charge $e = \sqrt{\pi^{-1} \cdot \frac{d^4}{t^3}}$ suggests that charge represents a specific orientation in rotational space. The concept of positive and negative charges corresponds to opposite phase orientations in the rotational dimensions.

This interpretation explains why charges come in discrete units—they represent complete rotational phase shifts in the two-dimensional rotational space. The conservation of charge becomes equivalent to the conservation of phase orientation in rotational dynamics.

4.2 Temperature as Rotational Frequency

With $k_B = \frac{d^5}{t^4 \pi}$, temperature emerges as a measure of rotational frequency squared. Thermal energy represents the fifth power of rotational displacement per fourth power of time, scaled by π.

This explains why temperature is fundamentally scalar rather than vector—it measures the magnitude of rotational frequency without directional components. Absolute zero corresponds to the cessation of rotational oscillation in the spatial dimensions.

4.3 Gravity as Rotational Dilution

Our expression for $G = \frac{1}{8\pi}$ reveals gravity as a consequence of rotational geometry rather than an independent force. The factor 8π suggests that gravitational interaction is related to the spatial dilution over a sphere in four-dimensional spacetime.

This geometric interpretation explains why gravity cannot be shielded or isolated—it represents a fundamental property of the rotational-temporal structure itself rather than a force mediated by particles.

4.4 Action as Volume Flux

With $\hbar = \pi^{-1} \cdot \frac{d^3}{t^2}$, quantum action represents a volume flux of rotational space per squared temporal progression. This explains why angular momentum is quantized in units of $\hbar$—it reflects the discrete nature of rotational volume elements in the $2+2$ dimensional structure.

The uncertainty principle then emerges as a natural consequence of the trade-off between rotational displacement and temporal progression, reflected in the d^3/t^2 dimensional structure of $\hbar$.

5 Cosmological Implications

5.1 Dark Energy Density

The cosmological constant, when dimensionally re-expressed:

$$\Lambda = 4\pi \cdot \frac{t^2}{d^2} = 4\pi \cdot \frac{t^2}{(2\pi t)^2} = \frac{1}{\pi} \tag{14}$$

This suggests that dark energy density is fundamentally related to the ratio of temporal to spatial dimensions. The accelerating expansion of the universe may represent a dynamic equilibration between the two temporal dimensions and two rotational spatial dimensions.

5.2 Cosmic Evolution

In the $2+2$ framework, cosmic evolution can be understood as the progressive dominance of temporal dimensions over spatial dimensions. This naturally explains the observed cosmic acceleration without requiring fine-tuned energy components.

The current state of the universe, with its specific ratio of dark energy to matter, may represent a transitional phase in this dimensional evolution rather than a coincidence requiring anthropic explanation.

5.3 Primordial Constants

Our dimensional framework suggests that fundamental constants may have evolved during cosmic history. At the earliest moments of the universe, when the distinction between dimensions was less pronounced, constants would approach their pure π-based values.

As the universe expanded and cooled, scale-dependent effects would cause constants to evolve toward their current measured values. This potentially explains apparent fine-tuning without requiring multiverse hypotheses.

6 Experimental Tests

6.1 Scale-Dependent Variations

Our framework predicts specific patterns of scale-dependent variation in fundamental constants:

1. The fine-structure constant α should approach $1/\pi$ at very high energies
2. The ratio $G\hbar c/\pi$ should remain exactly constant across all energy scales
3. The product $k_B T/(hc)$ should exhibit specific temperature dependence reflecting the underlying rotational dynamics

These predictions could be tested through precision measurements of constant variations across different energy scales and temperature regimes.

6.2 High-Energy Signatures

At energies approaching the Planck scale, our framework predicts distinctive signatures:

1. Particle interactions should reveal the fundamentally two-dimensional rotational nature of space
2. Cross-sections should show characteristic dependencies on π that become increasingly exact at higher energies
3. Quantum gravitational effects should manifest in ways that directly reflect the rotational-temporal coupling in the $2+2$ framework

Future high-energy collider experiments and astronomical observations of extreme environments could potentially test these predictions.

6.3 Quantum Optics Tests

Precision quantum optics experiments offer another avenue for testing our framework:

1. Interference patterns involving rotation in multiple planes should reveal subtle signatures of the two-dimensional rotational space
2. Phase-sensitive measurements might detect the geometric origin of the fine-structure constant as $1/\pi$
3. Quantum coherence effects could show distinctive dependencies on rotational orientation that reflect the underlying $2+2$ structure

7 Philosophical Significance

7.1 Constants as Emergent Rather Than Fundamental

Perhaps the most profound implication of our dimensional re-expression is that physical constants are not fundamental parameters requiring explanation or fine-tuning. Rather, they emerge necessarily from the rotational-temporal structure of reality.

With $d = 2\pi t$, all constants reduce to expressions involving pure powers of π. This suggests that the universe is deterministic at its deepest level—not in the sense of predicting specific events, but in the sense that its structural parameters are mathematically necessary rather than arbitrary.

7.2 Resolution of Fine-Tuning Problems

The apparent fine-tuning of constants for life—often cited as evidence for anthropic reasoning or a multiverse—finds a natural explanation in our framework. Constants are not independently adjustable parameters but different aspects of the same underlying rotational-temporal geometry.

This resolves the puzzle of why constants seem precisely calibrated to permit complex structures. They are not calibrated at all, but rather represent necessary geometric relationships in the $2+2$ dimensional structure.

7.3 Unification of Mathematics and Physics

Our framework suggests a deeper unity between mathematics and physics than previously recognized. The transcendental number π, traditionally viewed as a mathematical constant of circular geometry, emerges as the generator of all physical constants.

This suggests that the separation between mathematical structures and physical law may be an artifact of our limited understanding rather than a fundamental distinction. In the $2+2$ framework, physical law emerges directly from rotational geometry.

8 Conclusion

The dimensional re-expression of fundamental constants in terms of rotational displacement d, temporal progression t, and the geometric constant π reveals profound insights into the nature of physical law. By systematically tracking dimensions within the $2+2$ framework of Laursian Dimensionality Theory, we have demonstrated that:

1. All fundamental constants can be expressed as specific combinations of d, t, and π
2. These expressions reveal clear geometric interpretations for quantities traditionally viewed as arbitrary
3. With the substitution $d = 2\pi t$, all constants reduce to expressions involving pure powers of π
4. Physical constants are not independent, fine-tuned parameters but necessary consequences of rotational-temporal geometry

This extended π-System transforms our understanding of fundamental constants from arbitrary parameters that must be experimentally measured to necessary geometric relationships that emerge from the rotational-temporal structure of reality. It suggests that the universe may be simpler and more elegant at its foundation than conventional models indicate, with all apparent complexity emerging from the interplay of just two rotational spatial dimensions and two temporal dimensions.

While substantial experimental testing remains necessary to validate these dimensional interpretations, the mathematical consistency and explanatory power of the framework suggest that it captures essential aspects of physical reality. The ubiquitous appearance of π in these expressions is not coincidental but reflects the fundamentally rotational nature of the two spatial dimensions in Laursian Dimensionality Theory.

In this view, we arrive at a profound conclusion: constants are not fundamental—they are rotationally emergent. The universe's structural parameters are neither arbitrary nor fine-tuned, but rather necessary consequences of its rotational-temporal geometry.